

# Basic Kinematics Equations

$s = \frac{d}{t}$	$\bar{v} = \frac{\Delta x}{\Delta t}$	$\bar{v} = \frac{V_o + V}{2}$
$a = \frac{v - v_o}{t - t_o}$	$v \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$	$a \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$
$V_f = V_o + a\Delta t$	$V_f^2 = V_o^2 + 2ax$	$\Delta x = V_o\Delta t + \frac{1}{2}at^2$

Equations will be used when dealing with Problems with Constant Acceleration

# Derivations of the Equations of Motion

- The definition of acceleration is

$$a = \frac{v - v_o}{\Delta t}$$

- Multiply both sides by Delta t.

$$a\Delta t = v - v_o$$

- Add  $v_o$  to both sides.

$$v_o + a\Delta t = v$$

- This is our answer:

$$v_f = v_o + a\Delta t$$

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Velocity as a function of time.

$$v_f^2 = v_o^2 + 2ax$$

Velocity as a function of displacement.

$$\Delta x = v_o\Delta t + \frac{1}{2}at^2$$

Displacement as a function of time.

- Solve  $v = v_o + at$  for t.

$$t = \frac{v_f - v_o}{a}$$

- $v_{\text{bar}} = x/t$  thus  $x = v_{\text{bar}}$  times T.  
Substitute the above equation in for t in the equation  $x = v_{\text{bar}}$  times t.

$$x = v_{\text{bar}} \left( \frac{v_f - v_o}{a} \right)$$

- Substitute in the other equation for  $v_{\text{bar}}$ .

$$x = \left( \frac{v_f + v_o}{2} \right) \left( \frac{v_f - v_o}{a} \right)$$

- Multiply both sides by 2a.

$$2ax = (v_f + v_o)(v_f - v_o)$$

- Foil the right side.

$$2ax = v_f^2 - v_o^2$$

- Subtract  $v_o^2$  from both sides and we have our answer:

$$v_f^2 = v_o^2 + 2ax$$

- Set the Equations for V (bar) equal to another.

$$\frac{\Delta x}{\Delta t} = \frac{v_o + v_f}{2}$$

- Since  $v = v_o + at$  we can substitute the right term in place of v which removes a variable.

$$\frac{\Delta x}{\Delta t} = \frac{v_o + v_o + at}{2}$$

- We can combine like terms and break the right term into two separate terms each over 2.

$$\frac{\Delta x}{\Delta t} = \frac{2v_o}{2} + \frac{at}{2}$$

- The twos cancel and we can distribute the delta (t) by multiplying both sides by delta (t)

$$\Delta x = v_o\Delta t + \frac{1}{2}at^2$$